Multi-Strike and Multi-Strike 16X Video Poker Analysis

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This paper formally analyzes Multi-Strike and Multi-Strike 16X Video Poker games.

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1. Introduction

This paper explores a popular game by IGT[1] called Multi-Strike Video Poker and a variant called Multi-Strike 16X Video Poker. These games are also known as Multi-Rise Video Poker. We refer the reader to our earlier paper analyzing Ultimate X[2] Poker [1] for basic concepts.

Multi-Strike Video Poker is a sequential four-line game where one pays to play all four lines but can progress from one to the next only if they draw to a winning hand or are given a random Free Ride to the next line. After the draw of any hand, any winnings are paid a multiple of the normal payout depending on the level. The first level pays 1x the normal payout; level 2 pays 2x; level 3 pays 4x and level 4 pays 8x. Table 1 shows the normal payouts for one pay table available for the game Jacks or Better. Multi-Strike 16X adds one more line with a 16x multiplier.

Outcome	Per Coin
Royal Straight	**800**
Straight Flush	**50**
Four of a Kind	**25**
Full House	**9**
Flush	**6**
Straight	**4**
Three of a Kind	**3**
Two Pair	**2**
Jacks or Better	**1**
Nothing	**0**

Table 1: Jacks or Better 9-6 Pay Table

At the start of play, one places a max bet which is 20 times the selected coin denomination for Multi-Strike (and 25 times that for Multi-Strike 16X). The ”20” is 4 Lines of play times 5 coins per line (the max bet per line is 5 coins). So if the denomination is 25 cents, one would be betting $5. After placing the bet, 5 cards are randomly dealt from a standard deck of cards for the game (e.g., Joker Poker games would have 53 cards). The player then decides which cards to hold and to discard. Discarded cards are replaced by random draws from the remaining deck. If there is a winning outcome (or if a Free Ride card had momentarily been shown) the player proceeds to the

[1] Multi-Strike Video Poker games were created by IGT (https://www.igt.com/) and are offered in their video poker machines.
[2] Ultimate X was created by IGT (https://www.igt.com/) and is offered in their video poker machines.

next line after being paid for the win. Otherwise, the hand of play is over and the player must place a new bet or leave.

For example, using screen-shots from VideoPoker.com, here is the deal of the first hand of play. Holding the 2 Aces yielded a winning hand of play.

The draw ended with a Two Pair outcome, paying 10 coins (2 for each of the 5 coins bet per hand), The second hand received one Jack on the deal but got a second on the draw for a Jacks or Better outcome. Normally that pays 1 coin per coin bet, making 5 coins, but Line 2 has a 2x multiplier, so 10 coins were paid. Note in the following the total win is now 20 (10 from the first line and 10 from the second). And then play proceeds to the third line.

No cards were held on the third hand but the draw yielded a winning hand, a Pair of Kings, and was paid 4x the normal amount of 5 coins. The total winnings are now 40 coins.

Four of the cards were held for a possible Straight draw that didn't materialize. So this particular round was profitable yielding 40 coins on a 20 coin wager.

Occasionally, on the deal, a Free Ride card shows momentarily as in the following. In this case it wasn't necessary since the hand is already showing a winning Two Pair outcome (if held). The Free Ride automatically guarantees one progresses to the next line, even without a winning outcome. Normally, one might be more conservative on lower line hands, choosing holds with higher probabilities of wins over ones with lower probability but higher returns so one increases the chance of progressing to the next line. So getting a Free Ride is of value since knowing one will progress regardless of a winning hand allows a more aggressive strategy.

2. Expected Value Analysis

Let $\mathbb{H}$ be the set of all possible five card hands (order not a factor). For any five card hand, $H \in \mathbb{H}$, let H_i be the i$^{\text{th}}$ subset of H, i=0,…,31. For example, if $H = \{2H, JC, QD, 3S, 7S\}$, meaning a hand containing a 2 of Hearts, a Jack of Clubs, a Queen of Diamonds, a 3 of Spades and a 7 of Spades, then

$$H_0 = \{\ \}$$
$$H_1 = \{2H\}$$
$$H_2 = \{JC\}$$
$$H_3 = \{2H, JC\}$$
$$H_4 = \{QD\}$$
$$H_5 = \{2H, QD\}$$
$$H_6 = \{JC, QD\}$$
$$H_7 = \{2H, JC, QD\}$$
$$\ldots$$
$$H_{31} = \{2H, JC, QD, 3S, 7S\}$$

Let V_j be the value of outcome j relevant to the game (e.g., a Flush) and

$$P_j\left(H_i\right) = prob\Big(outcome_j \mid H_i\Big)$$

be the probability of outcome j computed from all possible completions of subset i from the deck of cards with the cards listed in H removed.

For a normal one-Line game, the expected return for choosing subset i is $R_{H_i} = \sum_j V_j P_j\left(H_i\right)$. The set of optimal actions, S_H, for hand $H \in \mathbb{H}$ is found by solving

$$S_H = \arg\max_i \sum_j V_j P_j\left(H_i\right).$$

Ties can be broken by choosing from optimizing actions based on other criteria (e.g., those minimizing or maximizing the variance as we do below). Multi-Strike is different in that one pays for 4-Lines of play but has to advance with wins or Free Rides to subsequent lines. So the expected return needs to consider future hands. For example, one might be more conservative on lower line hands, choosing holds with higher probabilities of wins over ones with lower probability but higher returns so one increases the chance of progressing to the next line. As mentioned above, the Free Ride feature complements these decisions since, if one has a Free Ride, a conservative play isn't needed.

If one makes it to Line-4 in Multi-Strike, one simply maximizes the expected return since no further hand depends on the outcome. So for any dealt hand H, one solves

$$8 \max_i R_{H_i}$$

And, when each hand is equally likely, the expected return is

$$EV_4 = \frac{8}{|\mathbb{H}|} \sum_{H \in \mathbb{H}} \max_i R_{H_i}$$

or just 8 times the optimal expected value of the normal underlying game. For 9-6 Jacks or Better, the optimal expected value is 0.99543904.

If one makes it to Line-3, they can get to Line-4 with a Free Ride or a winning outcome on hand 3. Let f_ℓ be the probability of a free ride on Line $\ell = 1, 2, \ldots$. Then the expected value of a hand H on Line-3 is

$$EV_3 = f_3\left(4EV + EV_4\right) + \frac{(1-f_3)}{|\mathbb{H}|}\sum_{H\in\mathbb{H}} \max_i\left(4R_{H_i} + \left(1-P_0\left(H_i\right)\right)EV_4\right).$$

$P_0\left(H_i\right)$ is the probability of no win. The first term gives the contribution given a Free Ride. Since one is guaranteed advancing to Line-4, the game's normal optimal expected value can be attained with a multiplier of 4x. The second term is for hands without a Free Ride. Note one gets 4 times the expected return plus the 4th line's return times the probability of advancing to the 4th line. Here it is clear how the decision on Line-3 impacts the overall return through the probability of a win $1-P_0\left(H_i\right)$.

In like manner, the expected value at Line-2 is

$$EV_2 = f_2\left(2EV + EV_3\right) + \frac{(1-f_2)}{|\mathbb{H}|}\sum_{H\in\mathbb{H}} \max_i\left(2R_{H_i} + \left(1-P_0\left(H_i\right)\right)EV_3\right)$$

and for Line-1

$$EV_1 = f_1\left(EV + EV_2\right) + \frac{(1-f_1)}{|\mathbb{H}|}\sum_{H\in\mathbb{H}} \max_i\left(R_{H_i} + \left(1-P_0\left(H_i\right)\right)EV_2\right).$$

Table 2 summarizes the games and pay tables that we are familiar with. We started with information available at [3].

Game	**Pay Table ID**	**EV**	**Min** σ^2	**Max** σ^2	**Free Ride Probabilities**		
Bonus Poker	8	99.3746	22.5578	22.5578	0.076	0.07	0.062
Bonus Poker	7	98.2245	22.3557	22.3557	0.076	0.07	0.062
Bonus Poker	6	97.08	22.2092	22.2092	0.076	0.07	0.062
Bonus Poker Deluxe	9-6	99.8631	33.1608	33.1608	0.078	0.072	0.062
Bonus Poker Deluxe	8-6	98.7106	32.9327	32.9327	0.078	0.072	0.062
Bonus Poker Deluxe	8-5	97.6931	33.4358	33.4358	0.078	0.072	0.062
Bonus Poker Deluxe	7-5	96.5396	33.2147	33.2147	0.078	0.072	0.062
Bonus Poker Deluxe	6-5	95.6476	33.7792	33.7792	0.078	0.072	0.062

Deuces Wild	25-15-9-5-3-2	100.881	26.3661	26.3679	0.0825	0.078	0.078
Deuces Wild	25-16-10-4-4-3	99.9203	26.3629	26.3629	0.09	0.081	0.079
Deuces Wild	25-15-9-4-4-3	98.9926	26.1242	26.1286	0.0891	0.0804	0.0775
Deuces Wild	20-12-10-4-4-3	97.8467	25.1952	25.1964	0.09	0.081	0.079
Deuces Wild	20-12-9-4-4-3	97.3749	25.325	25.325	0.09	0.081	0.079
Deuces Wild	25-16-13-4-3-2	97.2714	25.8941	25.8941	0.09	0.081	0.079
Deuces Wild	20-10-8-4-4-3	96.297	25.1715	25.1716	0.09	0.081	0.079
Deuces Wild	25-15-10-4-3-2	95.4939	25.8282	25.8282	0.09	0.081	0.079
Deuces Wild Bonus	9-4-3	99.5981	32.5706	32.5706	0.09	0.08	0.08
Deuces Wild Bonus	13-3-3	98.9937	31.9213	31.9213	0.09	0.08	0.0794
Deuces Wild Bonus	10-3-3	97.6926	32.5597	32.5605	0.09	0.08	0.08
Deuces Wild Bonus	12-3-2	96.7313	31.9725	31.9725	0.09	0.08	0.08
Deuces Wild Bonus	10-3-2	95.9198	32.242	32.242	0.09	0.08	0.08
Deuces Wild Dbl Bonus	25-12	99.894	39.6374	39.6374	0.0844	0.0805	0.0785
Deuces Wild Dbl Bonus	25-9	98.8195	39.7917	39.795	0.084	0.081	0.081
Deuces Wild Dbl Bonus	20-9	97.995	39.8448	39.8448	0.084	0.081	0.0802
Double Bonus	10-7-5	100.368	29.9976	29.9976	0.084	0.07	0.063
Double Bonus	9-7-5	99.2934	30.2733	30.2733	0.084	0.07	0.063
Double Bonus	9-6-5	97.9986	31.7067	31.7067	0.083	0.069	0.062
Double Bonus	9-6-4	97.0592	31.7279	31.7279	0.084	0.07	0.063
Double Bonus	8-5-4	94.9394	31.7691	31.7691	0.084	0.07	0.063
Double Dbl Bonus	10-6	100.258	42.3767	42.3767	0.079	0.07	0.061
Double Dbl Bonus	9-6	99.1788	42.1569	42.1569	0.079	0.07	0.061

Double Dbl Bonus	9-5	97.9948	42.417	42.417	0.077	0.07	0.061
Double Dbl Bonus	8-5	97.0694	42.2804	42.2804	0.079	0.07	0.061
Double Dbl Bonus	7-5	96.002	42.3411	42.3411	0.079	0.07	0.061
Double Dbl Bonus	6-5	94.9553	42.2716	42.2716	0.079	0.07	0.061
Jacks or Better	9-6	99.7918	20.9959	20.9959	0.077	0.07	0.064
Jacks or Better	9-5	98.781	21.211	21.211	0.077	0.07	0.064
Jacks or Better	8-5	97.6302	20.9893	20.9893	0.077	0.07	0.064
Jacks or Better	7-5	96.48	20.7864	20.7864	0.077	0.07	0.064
Jacks or Better	6-5	95.3302	20.6184	20.6184	0.077	0.07	0.064
Joker Poker (2 Pair)	1000-100-50-50-20-10-6-5	99.9993	30.8337	30.8342	0.282	0.281	0.277
Joker Poker (2 Pair)	100-800-100-100-16-8-5-4	97.5241	69.2194	69.2194	0.284	0.28	0.278
Joker Poker (Aces)	1000-200-100-50-20-6-5	94.4944	34.5519	34.5519	0.186	0.18	0.176
Joker Poker (Kings)	800-50-20-7-5	100.832	26.2434	26.2434	0.089	0.081	0.077
Joker Poker (Kings)	940-50-17-7-5	98.6311	31.5118	31.5118	0.089	0.082	0.079
Joker Poker (Kings)	940-50-15-7-5	96.9436	30.9895	30.9895	0.089	0.082	0.081
Joker Poker (Kings)	800-50-15-7-5	96.5837	25.098	25.098	0.088	0.082	0.082
Joker Poker (Kings)	800-40-20-5-4	95.694	24.7802	24.7823	0.086	0.08	0.079
Super Aces Bonus	8	99.9963	62.3038	62.3038	0.0792	0.0673	0.0587
Super Aces Bonus	7	98.9923	62.2252	62.2252	0.08	0.068	0.0576
Super Aces Bonus	6	97.9927	62.334	62.334	0.08	0.068	0.0597
Super Double	9	99.885	39.0379	39.0379	0.078	0.068	0.06
Super Double	8	98.8892	39.7277	39.7312	0.078	0.068	0.06

Super Double	7	97.9902	39.595	39.595	0.078	0.068	0.06
Super Double	6	97.1089	39.5025	39.5025	0.078	0.068	0.06
Super Dbl Dbl	8	99.8932	51.2467	51.2467	0.077	0.068	0.061
Super Dbl Dbl	7	98.832	51.2261	51.2261	0.077	0.068	0.061
Super Dbl Dbl	6	97.9348	51.6	51.6	0.077	0.068	0.061
Triple Dbl Bonus	9-7	99.7648	95.8845	95.8845	0.0835	0.066	0.059
Triple Dbl Bonus	9-6	98.5777	97.2489	97.2489	0.0835	0.066	0.059
Triple Dbl Bonus	9-5	97.5285	97.8771	97.8786	0.0835	0.066	0.059
Triple Dbl Bonus	8-5	96.4929	97.9268	97.9269	0.0835	0.066	0.059
Triple Dbl Bonus	7-5	95.4591	97.9063	97.9063	0.0835	0.066	0.059
White Hot Aces	9	99.7679	43.933	43.933	0.079	0.069	0.062
White Hot Aces	8	98.6967	44.0026	44.0026	0.079	0.069	0.062
White Hot Aces	7	97.649	43.8639	43.8639	0.079	0.069	0.062
White Hot Aces	6	96.6037	43.7125	43.7125	0.079	0.069	0.062

Table 2: Multi-Strike Analyses. For each game/pay table combination, we show the per-coin expected value of the game, the per-coin max/min variance of the game, and the Free Ride probabilities used.

Notice the variance of 20.9959 in 9-6 Jacks or Better. The variance for regular 9-6 Jacks or better is 19.5147, which may seem odd given one is wagering more in Multi-Strike. Dr. Rick Percy provided a great explanation on the WizardOfVegas blog [2]:

> "For anyone surprised that 20.9959 is not that much bigger than the variance associated with regular 9-6 Jacks (19.5147), it is helpful to remember that the units here in both cases are bets-squared. Since the MultiStrike bet is 20 coins instead of the standard 5-coin bet, we can show the two variances in a different light in units of coins-squared to be 487.8669 (19.5147 x 5^2) for standard Jacks vs. 8398.3535 (20.9959 x 20^2) for MultiStrike Jacks."

In Multi-Strike 16, if one makes it to line 5 they get

$$EV_5 = \frac{16}{|\mathbb{H}|} \sum_{H \in \mathbb{H}} \max_i R_{H_i} \ .$$

Similarly, Line 4's expected value is:

$$EV_4 = f_4\left(8EV + EV_5\right) + \frac{(1-f_4)}{|\mathbb{H}|} \sum_{H \in \mathbb{H}} \max_i \left(8R_{H_i} + \left(1 - P_0\left(H_i\right)\right) EV_5\right)$$

and so forth as above. Table 3 summarizes the Multi-Strike 16X games and pay tables that we are familiar with. We started with information available at [4]. We will update whenever we acquire additional Free Ride probabilities.

3. Variance Analysis

An early version of this paper derived an equation for the variance and argued that Levels were independent so covariance terms could be ignored. An astute reader, Dr. Rick Percy [2], pointed out:

> "For simplicity, if we just consider the random variable that is the sum of the Level 3 outcome plus the Level 4 outcome, for any Level 3 outcome other than zero, it is true that all the Level 4 outcome probabilities are the same. But if the Level 3 outcome happens to be zero, you can predict that the Level 4 outcome will also be zero more than 97% of the time. So, the random variable representing the Level 3 outcome is clearly not independent of the random variable representing the Level 4 outcome, so you cannot do a handwave and ignore the covariances entirely.".

He was correct. However, deriving a nice expression for the variances is not trivial when covariances are involved. Instead, we computed variances shown in Tables 2 and 3 using a standard approach. We enumerated all the possible totals (call these X) and their probabilities, and then computed the variance using the usual

$$Var(X) = E\left(X^2\right) - E(X)^2 .$$

As mentioned earlier, expected value ties for different holds can be broken by choosing from optimizing actions based on other criteria, such as optimizing holds with minimal or maximal variance. The resulting overall min and max variances are shown in Tables 2 and 3. In most games, there aren't ties (or enough to make a difference), but in some there is a notable difference in the minimum and maximum variances.

Game	Pay Table ID	EV	Min σ^2	Max σ^2	Free Ride Probabilities			
Bonus Poker	8	99.4975	66.3875	66.3875	0.076	0.07	0.062	0.0575
Bonus Poker	7	98.3556	65.7612	65.7612	0.076	0.07	0.062	0.0575
Bonus Poker	6	97.22	65.404	65.404	0.076	0.07	0.062	0.0575
Bonus Poker Deluxe	9-6	99.8932	78.9456	78.9456	0.078	0.072	0.062	0.0575
Bonus Poker Deluxe	8-6	98.7484	78.6387	78.6387	0.078	0.072	0.062	0.0575
Bonus Poker Deluxe	8-5	97.7436	79.8147	79.8147	0.078	0.072	0.062	0.0575
Bonus Poker Deluxe	7-5	96.5959	79.4239	79.4239	0.078	0.072	0.062	0.0575
Bonus Poker Deluxe	6-5	95.7142	85.7834	85.7834	0.078	0.072	0.062	0.0575
Deuces Wild	25-16-10-4-3	99.9486	81.7671	81.7671	0.0891	0.0825	0.078	0.0754
Deuces Wild	25-15-9-4-3	98.9827	81.2201	81.2399	0.0891	0.0804	0.0768	0.0754
Deuces Wild	20-12-10-4-3	97.8831	78.2574	78.261	0.09	0.081	0.079	0.074
Deuces Wild	20-12-9-4-3	97.4185	78.7898	78.7898	0.09	0.081	0.079	0.074
Deuces Wild	25-16-13-3-2	97.3045	80.2568	80.2568	0.09	0.081	0.079	0.074
Deuces Wild	20-10-8-4-3	96.3427	78.2648	78.2654	0.09	0.081	0.079	0.074
Deuces Wild	25-15-10-3-2	95.5489	79.9816	79.9816	0.09	0.081	0.079	0.074
DW Bonus	9-4-3	99.6494	127.013	127.013	0.09	0.08	0.08	0.076
DW Bonus	13-3-3	98.986	124.109	124.109	0.09	0.08	0.0787	0.076
DW Bonus	10-3-3	97.7477	126.872	126.88	0.09	0.08	0.08	0.076
DW Bonus	12-3-2	96.7979	124.226	124.226	0.09	0.08	0.08	0.076
DW Bonus	10-3-2	95.9973	125.967	125.967	0.09	0.08	0.08	0.076
DW Dbl Bonus	25-12	100.008	154.385	154.385	0.084	0.081	0.081	0.075
DW Dbl Bonus	25-9	98.8812	155.894	155.914	0.084	0.081	0.081	0.075
DW Dbl Bonus	20-9	98.1008	156.794	156.794	0.084	0.081	0.081	0.075
Double Bonus	9-7-5	99.3263	85.8547	85.8547	0.084	0.07	0.063	0.0535
Double Bonus	9-6-5	97.9974	91.643	91.643	0.083	0.069	0.0614	0.0535
Double Bonus	9-6-4	97.1525	91.2895	91.2895	0.084	0.07	0.063	0.0535
Double Bonus	8-5-4	95.0525	91.2968	91.2968	0.084	0.07	0.063	0.0535
Dbl Dbl Bonus	9-6	99.1795	138.915	138.915	0.079	0.07	0.061	0.0515
Dbl Dbl Bonus	9-5	97.9958	140.005	140.005	0.077	0.07	0.0608	0.0515
Dbl Dbl Bonus	8-5	97.1142	139.606	139.606	0.079	0.07	0.061	0.0523
Dbl Dbl Bonus	7-5	96.0549	140.089	140.089	0.079	0.07	0.061	0.0523

Dbl Dbl Bonus	6-5	95.0164	139.856	139.856	0.079	0.07	0.061	0.0523
Jacks or Better	9-6	99.8118	53.062	53.062	0.077	0.07	0.064	0.0524
Jacks or Better	9-5	98.8187	53.3196	53.3196	0.077	0.07	0.064	0.0524
Jacks or Better	8-5	97.6784	52.7057	52.7057	0.077	0.07	0.064	0.0524
Jacks or Better	7-5	96.5387	52.4118	52.4118	0.077	0.07	0.064	0.0524
Jacks or Better	6-5	95.3994	52.1285	52.1285	0.077	0.07	0.064	0.0524
Joker Poker (K)	940-17	98.6552	71.4537	71.4537	0.089	0.082	0.079	0.075
Joker Poker (K)	940-15	96.9915	70.0014	70.0014	0.089	0.082	0.081	0.075
Joker Poker (K)	800-15	96.6524	56.6597	56.6597	0.088	0.082	0.082	0.075
Super Aces Bns	7	98.9984	183.344	183.344	0.08	0.068	0.0575	0.0499
Super Aces Bns	6	97.9526	183.759	183.759	0.08	0.068	0.0575	0.0499
Super Double	9	99.903	120.903	120.903	0.078	0.068	0.06	0.05
Super Double	8	98.9184	128.351	128.368	0.078	0.068	0.06	0.05
Super Double	7	98.032	127.917	127.917	0.078	0.068	0.06	0.05
Super Double	6	97.1638	127.501	127.501	0.078	0.068	0.06	0.05
Triple Dbl Bonus	9-7	99.7829	319.972	319.972	0.0835	0.066	0.059	0.0508
Triple Dbl Bonus	9-6	98.6309	325.345	325.345	0.0835	0.066	0.059	0.0508
Triple Dbl Bonus	9-5	97.6032	327.888	327.896	0.0835	0.066	0.059	0.0508
Triple Dbl Bonus	8-5	96.5754	328.818	328.818	0.0835	0.066	0.059	0.0508
Triple Dbl Bonus	7-5	95.549	330.299	330.299	0.0835	0.066	0.059	0.0508

Table 3: Multi-Strike 16 Analyses. For each game/pay table combination, we show the per-coin expected value of the game, the per-coin max/min variance of the game, and the Free Ride probabilities used.

4. Acknowledgements

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Also, we appreciate the many e-mail discussions with Michael Shackleford, The Wizard of Odds.

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