Fortune X Poker Analysis

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This paper analyzes a new game by IGT[1] using a non-discounted, infinite horizon, discrete time Markov Decision problem.

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[1] https://www.igt.com/

1. Expected Value Computation

Fortune X by IGT offers various video poker games at the usual payoffs if only 5 coins per line are bet. If 10 coins per line are bet, the Fortune X Multiplier feature is eligible. If eligible the Fortune X feature may be awarded after the deal. The feature is awarded an average of 11% (i.e., here we use $f = 0.11$) for the 98.98008% Double Double Bonus game and starts with a multiplier of 2. A player may elect to apply the multiplier on the outcome of the dealt hand or reject using the multiplier and play the hand as usual (i.e., with just a unit multiplier). If he uses the multiplier any result will be multiplied by the current multiplier, m, and m is reset to one for the next hand. If not, the next hands multiplier will increase, using the progression 2, 3, 5, 8, 12. Once 12 is reached, the multiplier will be applied automatically to the hand's outcome. So, once the feature is active with a multiplier of 2, 3, 5 or 8, a decision is made after receiving the dealt hand on whether to accept the current multiplier or reject it. The possible states of the multipliers are contained in the set $S = \{1, 2, 3, 5, 8, 12\}$ with decisions required at states in $M = \{2, 3, 5, 8\}$.

Note, there seems to be some confusion on what f pertains to. I interpret this value as the percentage across all hands played at the 10 coin max bet basis. That means that the probability going from a non-rewarded multiplier state to the start of rewards is actually

$$\frac{f}{1-f}.$$

The following two conjectures are trivial to prove.

Conjecture 1

If the optimal decision for dealt hand h and multiplier $m \in M$ is to accept the multiplier, then any other potential dealt hand with an expected value greater than or equal to h's expected value is to also accept the multiplier.

Conjecture 2

If the optimal decision for dealt hand h and multiplier $m \in M$ is to accept the multiplier, then the optimal decision for h and a multiplier greater than m is also to accept the multiplier.

Given Conjectures 1 and 2, the decisions on whether to use a multiplier $m \in M$ for any hand can be simplified to determining if the hand's expected value is greater than or equal to a value b_m. If so, m is used. Otherwise, the hand is played as usual for the base game and the next hand will have the next multiplier in the progression. So there are just four decision variables.

For any video poker game one can compute the number of unique hands for the given base game (e.g., for Double Double Bonus, there are 134,459 unique hands). However, each such game has far fewer unique expected values. For example, 98.98% Double Double Bonus has only 1,210 unique expected values. So, at worst, enumerating all the possible settings for the four b_m values would take a little over 4 trillion combinations (i.e., 1211^4, the extra one is for adding the case where no hands will accept a multiplier). However, conjecture 2 shows we can reduce that since it implies $0 \le b_2 \le b_3 \le b_5 \le b_8$. Hence, only 89,759,762,016 cases need to be evaluated. Additional shortcuts are possible.

Let R be the expected EV of the base game played optimally and $r(b)$ be the cumulative EV of all dealt hands played optimally having a value at least equal to b. Likewise, let $p(b)$ be the cumulative probability of these hands. The transition matrix across for the states S is

$$P(b_2,b_3,b_5,b_8)=\begin{pmatrix} 1-\dfrac{f}{1-f} & \dfrac{f}{1-f} & 0 & 0 & 0 & 0 \\ p(b_2) & 0 & 1-p(b_2) & 0 & 0 & 0 \\ p(b_3) & 0 & 0 & 1-p(b_3) & 0 & 0 \\ p(b_5) & 0 & & 0 & 1-p(b_5) & 0 \\ p(b_8) & 0 & 0 & 0 & 0 & 1-p(b_8) \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

and reward vector

$$r\left(b_2,b_3,b_5,b_8\right)=0.5\begin{pmatrix} R \\ 2r\left(b_2\right)+R-r\left(b_2\right) \\ 3r\left(b_3\right)+R-r\left(b_3\right) \\ 5r\left(b_5\right)+R-r\left(b_5\right) \\ 8r\left(b_8\right)+R-r\left(b_8\right) \\ 12R \end{pmatrix}=0.5\begin{pmatrix} R \\ r\left(b_2\right)+R \\ 2r\left(b_3\right)+R \\ 4r\left(b_5\right)+R \\ 7r\left(b_8\right)+R \\ 12R \end{pmatrix}.$$

Multiplying by 0.5 is necessary since the bet size is twice the usual 5 coin max bet and the EVs are on a per coin-in basis. The Markov property applies to these games, the chain is ergodic and, assuming an infinite horizon without discounting, one wishes to find the maximum gain, g, per hand by solving

$$v+ge=\max_{0\le b_2\le b_3\le b_5\le b_8} r\left(b_2,b_3,b_5,b_8\right)+P\left(b_2,b_3,b_5,b_8\right)v$$
$$\pi\left(b_2,b_3,b_5,b_8\right)'v=0$$

where e is a vector of ones, v is a vector of state biases, and $\pi\left(b_2,b_3,b_5,b_8\right)$ is the steady-state probabilities vector satisfying

$$\pi\left(b_2,b_3,b_5,b_8\right)'P\left(b_2,b_3,b_5,b_8\right)=\pi\left(b_2,b_3,b_5,b_8\right)'$$

where x' is the transpose of any vector x. One can simplify the above to

$$\max_{0\le b_2\le b_3\le b_5\le b_8} e_1'\begin{pmatrix} e & I-P\left(b_2,b_3,b_5,b_8\right) \\ 0 & e_6' \end{pmatrix}^{-1}\begin{pmatrix} r\left(b_2,b_3,b_5,b_8\right) \\ 0 \end{pmatrix}$$

where I is the identity matrix and e_i is the i[th] unit vector. Using e_6' or any other in the above matrix just alters the biases but does not change g. For $f=0.11$ and the Double Double Bonus poker, where $R=0.98980783486934$, the optimal gain is 0.990069802845 with optimal values

$$b_2=9$$
$$b_3=5.36632747456$$
$$b_5=3.53191489362$$
$$b_8=1.42183163737$$

For 9-6 Jacks or better with $f = 0.1165$ where $R = 0.99543904369511$, the optimal gain is 0.995994710061 with optimal values

$$b_2 = 9$$
$$b_3 = 6$$
$$b_5 = 2.59574468085$$
$$b_8 = 1.43293246994$$

2. Variance Computation

Once the optimal b-values are known, we enumerated all the possible hand totals (call these X) and their probabilities, and then computed the variance using the usual

$$Var\left(X\right) = E\left(X^2\right) - E\left(X\right)^2.$$

The Double Double Bonus poker's variance is 142.5663773 and the Jacks or Better variance is 65.45201477.

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